Analyticity and the N_c counting rule of S matrix poles

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By studying $\pi\pi$ scattering amplitudes in the large N_c limit, we clarify the N_c dependence of the S matrix pole position. It is demonstrated that analyticity and the N_c counting rule exclude the existence of S matrix poles with $\mathcal{M}$, $\Gamma \sim O(1)$. Especially the properties of σ and $f_0(980)$ with respect to the $1/N_c$ expansion are discussed. We point out that in general tetra-quark resonances do not exist.

Recently there are increased interests in investigating the nature of the $f_0(600)$ or the σ resonance, which is important for a deeper understanding of spontaneous chiral symmetry breaking of QCD. Also there are revived interests in recent literature to search for exotic states, for example, the tetra quark states appear in meson-meson scatterings. In this Letter we devote to the study of these problems, using techniques from the S matrix theory, low energy effective theory and large N_c expansion. Considering the difficulty of the problem, we will mainly confine ourselves in dealing with elastic $\pi\pi$ scattering amplitudes.

We begin by noticing that, for the partial wave elastic scattering, the physical S matrix can in general be factorized as, [1, 2]

$$S^{phy} = \prod_i S^{p,i} \cdot S^{cut} , \qquad (1)$$

where $S^{p,i}$ are the simplest S matrices characterizing isolated singularities of S^{phy}, that is, for virtual/bound states: $S(s) = (1 \pm i\rho(s)a)/(1 \mp i\rho(s)a)$ and for a resonance located at z_0 (z_0^*) on the second sheet: $S^R(s) = (M^2[z_0] - s + i\rho(s)s\,G[z_0])/(M^2[z_0] - s - i\rho(s)s\,G[z_0])$ where

$$M^2[z_0] = \text{Re}[z_0] + \text{Im}[z_0]\frac{\text{Im}[\sqrt{z_0(z_0 - 4m_\pi^2)}]}{\text{Re}[\sqrt{z_0(z_0 - 4m_\pi^2)}]} ,$$

$$G[z_0] = \frac{\text{Im}[z_0]}{\text{Re}[\sqrt{z_0(z_0 - 4m_\pi^2)}]} . \qquad (2)$$

The functions $M^2[z_0]$ and $G[z_0]$ have the properties that if $\text{Im}[z_0] \neq 0$ then either $M^2[z_0] > 4m_\pi^2$, $G[z_0] > 0$, or $M^2[z_0] < 0$, $G[z_0] < 0$. These properties will be useful later. The pole mass and width are denoted as $z_0 \equiv (\mathcal{M} + i\Gamma/2)^2$ in this Letter. For the reason which will become apparent later, parameters M^2 and G (or $\text{Re}[z_0]$, $\text{Im}[z_0]$) more appropriately describe a resonance than $\mathcal{M}$ and Γ. For a narrow resonance located in the region detectable experimentally we have approximately $\mathcal{M} = M$, $G = \Gamma/M$. It is also worth noticing that the resonance and also the virtual state contributions to the scattering length and phase shift are always positive whereas the bound state pole contribution is always negative.

The S^{cut} contains only cuts which can be parameterized in the following simple form, [2]

$$S^{cut} = e^{2i\rho f(s)} , \qquad (3)$$

$$f(s) = \frac{s}{\pi} \int_L \frac{\text{Im}_L f(s')}{s'(s'-s)} ds' + \frac{s}{\pi} \int_R \frac{\text{Im}_R f(s')}{s'(s'-s)} ds'$$

$$\equiv f_L(s) + f_R(s) . \qquad (4)$$

where $L = (-\infty, 0]$ and R denotes cuts at higher energies other than the 2π elastic cut. It starts at 4π threshold but to a good approximation it starts at $4m_K^2$. The discontinuity f obeys the following simple relation:

$$\text{Im}_{L,R} f(s) = -\frac{1}{2\rho(s)} \log |S^{phy}(s)|$$

$$= -\frac{1}{4\rho} \log \left[1 - 4\rho\text{Im}_{L,R}T + 4\rho^2|T(s)|^2\right] . \quad (5)$$

Before proceeding it is important to notice that Eqs. (4) and (5) are obtained by assuming that the partial wave amplitudes are analytic on the whole cut plane, which can be derived, for example, by assuming the validity of Mandelstam representation for the full T matrix amplitude. Nevertheless the validity of Mandelstam representation goes beyond what is rigorously established from field theory, and what is rigorously established on analyticity of partial wave amplitudes is the Lehmann–Martin domain of analyticity. [3] If Eq. (4) becomes invalid, it should be replaced by the following expression:

$$f(s) = \frac{s}{2\pi i} \int_C \frac{f_C(s')}{s'(s'-s)} ds' + \frac{s}{\pi} \int_R \frac{\text{Im}_R f(s')}{s'(s'-s)} ds' ,$$

$$f_C = \frac{1}{2i\rho} \log \left(1 + 2i\rho T^{phy}\right) , \qquad (4')$$

where C is the contour separating the Lehmann–Martin domain and the region unknown. In the following we will assume the validity of Eq. (4) but our major conclusions on the N_c properties of resonance states depend very little on Eq. (4).

Starting from Eq. (5), the first observation is, the integrand of the left hand dispersion integral for function f is not allowed to make a chiral perturbation expansion, due to the $1/\sqrt{s}$ singularity hidden in the relativistic kinematic factor $\rho(s)$. This phenomenon does not necessarily lead to

any profound impact on the validity of chiral expansions. This may be best illustrated by the following example, the integral

$$\frac{1}{2\pi}\int_0^\infty \frac{\log(1+\alpha^2/x)}{\sqrt{x}(1+x)}dx = \log(1+|\alpha|)$$
$$= \alpha - \frac{\alpha^2}{2} + \frac{\alpha^3}{3} + \cdots, \quad (\alpha > 0) \tag{6}$$

does not allow an expansion on the integrand in powers of the coupling constant α, but after performing the integration it can still be expanded in powers of α (though the expansion is not analytic at $\alpha = 0$). A related problem is the N_c power counting of f_L. Since $T \sim O(N_c^{-1})$ when making the large N_c expansion one may naively neglect the the term proportional to $|T|^2$ inside the logarithm on the r.h.s. of the second equality of Eq. (5) since it is $O(N_c^{-2})$, but Eq. (6) reveals that it will come back and make a contribution at $O(N_c^{-1})$. On the other side, χPT predicts $\mathrm{Im}_L T$ to be $O(N_c^{-2})$, but at higher energies it is no longer true. We in fact have $\mathrm{Im}_L T \sim O(N_c^{-1})$ from further left hand cuts contributed by crossed channel resonance exchanges. To see this recall that, [4]

$$\mathrm{Im}\, T_l^I(s) = \frac{[1+(-1)^{l+I}]}{s-4}\sum_{l'}\sum_{I'}(2l'+1)C_{II'}^{(st)}$$
$$\times \int_4^{4-s} dt\, P_l(1+\frac{2t}{s-4})P_{l'}(1+\frac{2s}{t-4})\mathrm{Im}\, T_{l'}^{I'}(t), \tag{7}$$

which relates the nearby left hand cut to the physical region singularities. Using Eq. (7) together with narrow resonance approximation $\mathrm{Im}T(t) \sim \pi\sum\Gamma_i M_i\delta(M_i^2 - t)$ one finds indeed that the left cut is $O(N_c^{-1})$. [5]

A natural way to avoid the problem of the expansion at $s = 0$ as mentioned above is to make instead a threshold expansion on f, $f = \sum_{n=0}^\infty f_n\frac{(s-4m_\pi^2)^n}{(m_\pi^2)^n}$. The contributions from the left hand and right hand integral to each coefficient will be denoted as f_{Ln} and f_{Rn}, respectively. In the large N_c limit f_L can be written as:

$$f_L(s) = \frac{s}{\pi}\int_L \frac{\mathrm{Im}_L T(s')}{s'(s'-s)}ds' - |T(0)| + O(N_c^{-2}), \tag{8}$$

where the $O(N_c^{-2})$ part of $\mathrm{Im}_L T$ and $T(0)$ are to be neglected. Here we will not attempt to calculate the left cut explicitly. What is really important to us is that $\mathrm{Im}_L T$ (and hence f_{Ln}) is $O(N_c^{-1})$, it certainly can not be $O(1)$ since T itself is $O(N_c^{-1})$. Another important fact of Eq. (8) is that it makes sense to make a low energy expansion to the integral, since the integration starts effectively from $4m_\pi^2 - M^2$ (where M is the mass of the lightest crossed channel resonance), rather than from 0.

In Eq. (1) only second sheet poles are explicitly parameterized and poles on other sheets are all hidden in f_R. For the latter, it is naively expected to be also of $O(1/N_c^2)$. But since there are poles on sheets closely connected with the physical region approaching the upper half of the physical cut (for example in $\pi\pi$, $\bar{K}K$ couple channel system, narrow poles on the third sheet, but not on the fourth sheet). In the large N_c limit, the integration path will come over those poles and pinched singularities will occur. The N_c order will also change in this case. When only these poles are considered, using parametrization [6]

$$T_{1n} = \frac{1}{\sqrt{\rho_1(s)\rho_n(s)}}\sum_r \frac{\mathcal{M}_r(\Gamma_{r1}\Gamma_{rn})^{\frac{1}{2}}}{\mathcal{M}_r^2 - s - i\mathcal{M}_r\Gamma_r} + C, \tag{9}$$

with Γ_{rn} the partial width, Γ_r the total width and C the smooth background at most of $O(N_c^{-1})$, and taking, for example, $\Gamma_r \sim O(1/N_c)$, the r.h.c. integral can be carried out:

$$\frac{s}{\pi}\int_R \frac{\mathrm{Im}_R f(s')}{s'(s'-s)}ds' = \sum_r \frac{G_r s}{\mathcal{M}_r^2 - s}, \tag{10}$$

which holds in the large N_c limit when $s \ll M_r^2$ and $G_r = \frac{\Gamma_r(1-\sqrt{1-\alpha_r})}{2\sqrt{(\mathcal{M}_r^2-4m_\pi^2)}}$, where $\alpha_r = 4\Gamma_{r1}(\Gamma_r - \Gamma_{r1})/\Gamma_r^2$. G_r is distinguished from G of the second sheet resonances by subscript r. Now f_{R0}, f_{R1}, f_{R2} can be estimated,

$$f_{R0} = \sum_r \frac{4m_\pi^2 G_r}{\mathcal{M}_r^2 - 4m_\pi^2}, \quad f_{R1} = \sum_r \frac{\mathcal{M}_r^2 m_\pi^2 G_r}{(\mathcal{M}_r^2 - 4m_\pi^2)^2},$$
$$f_{R2} = \sum_r \frac{\mathcal{M}_r^2 m_\pi^4 G_r}{(\mathcal{M}_r^2 - 4m_\pi^2)^3}, \tag{11}$$

where higher order terms of $1/N_c$ expansion are neglected. From Eq. (10) we find that after integration the higher sheet poles contributions are really of $O(1/N_c)$. In the definition of G_r, since $\mathcal{M}_r > 4m_K^2$ if neglecting the 4π cut, and by definition $\Gamma_r > \Gamma_{r1}$, G_r is positive. From Eq. (11) we can see that f_{R0}, f_{R1}, f_{R2} are all positive (which can actually be directly obtained from positivity of $\mathrm{Im}_R T$). It should be realized that if there exists a higher sheet pole with $\Gamma_r \sim O(1)$, $M_r^2 \sim O(1)$, i.e., not approaching real axis, then such a pole does not enter into Eq. (11). The effect of such a pole can only be cancelled by a nearby zero (i.e., a pole on other sheets). It cannot be cancelled by a nearby pole on the same sheet because the other pole has a negative norm (like the time-like component of the photon field) in order to make the cancellation take place and to make the elastic $\pi\pi$ scattering amplitude $O(1/N_c)$ (since they do not approach real axis, the net effect to Eq. (11) after the cancellation is $1/N_c^2$ suppressed). However, a pole with negative norm causes the severe problem of negative probability and hence should not appear. Also it should be mentioned that here we can not exclude, by only looking at the N_c order of $T_{\pi\pi\to\pi\pi}$ amplitude, a higher sheet pole like, $\Gamma \sim O(1)$, $\mathcal{M} \sim O(1)$ but $\Gamma_{r1} \sim O(1/N_c^2)$. The existence of such a resonance does not contradict the N_c counting rule of $T_{\pi\pi\to\pi\pi}$. The problem may only be studied by analyzing the, for example, $T_{\bar{K}K\to\bar{K}K}$ amplitude. Related discussions will be given later.

According to the conventional wisdom, the complete S matrix defined in Eq. (1) can be faithfully parameterized by the low energy effective theory, i.e., $S^{phy} = S^{\chi PT}$, in a limited low energy region on the complex s plane. Implicitly the above statement requires that there is a convergence radius for the low energy theory which do not shrinks to zero for arbitrary value of N_c. This is necessary because otherwise we can not make any expansion. Also it requires that there is no bound state pole for $\pi\pi$ scatterings in N_c QCD, since inside the validity domain, the physical spectrum as predicted by the low energy theory should be respected. The condition on the absence of bound state pole is not absolutely necessary for our later purpose, though it will simplify our discussion considerably . What we will do in the following is to make a threshold expansion to the S matrices of resonance poles, $S^p \equiv \prod_i S^{p,i}$, on the $r.h.s.$ of Eq. (1). For the present purpose we recast Eq. (1) as,

$$S^p(s) = S^{phy}(S^{cut})^{-1} = S^{phy}e^{-2i\rho\,f(s)}\,. \qquad (12)$$

The matching between the $l.h.s.$ and the $r.h.s.$ of the above equation can be performed at sufficiently small energies if we make a threshold as well as a chiral expansion on S^{phy}, that is to replace S^{phy} on the $r.h.s.$ of the above equation by $S^{\chi PT}$, for the latter we have the standard N_c counting rules [7]. It has been illustrated that one can make a low energy expansion on $f(s)$ on the $r.h.s.$ of Eq. (12) as well, with each coefficients at most $O(N_c^{-1})$. Therefore the matching between the $l.h.s.$ and the $r.h.s.$ of Eq. (12) can be done in the leading order of $1/N_c$ expansion. $S^p(s)$ can be safely expanded at the $\pi\pi$ threshold, since for any resonance pole not lying on the real axis there is always $M^2[z_0] \neq 4m_\pi^2$. For simplicity of discussions we did not include virtual poles, but no conclusion will be changed if they are included. It is straightforward to demonstrate the following relation,

$$\frac{(S^p(s)-1)}{2i\rho(s)} = \sum_i \frac{4G_i m_\pi^2}{M_i^2-4m_\pi^2} + \sum_i \frac{G_i M_i^2}{(M_i^2-4m_\pi^2)^2}$$
$$\times(s-4m_\pi^2) + \sum_i \frac{G_i M_i^2}{(M_i^2-4m_\pi^2)^3}(s-4m_\pi^2)^2 + \cdots$$
$$+O(N_c^{-2})\,. \qquad (13)$$

Every coefficient of the series on the $r.h.s.$ of the above equation is $O(N_c^{-1})$. Since there are no bound state poles by assumption (virtual state poles are harmless), then the only isolated singularities appeared here are the second sheet resonances. Then according to the properties of M_i^2 and G_i, every term in the first (and also the second, but not the third) coefficient is positive. Therefore *the N_c order of each $G_i/(M_i^2 - 4m_\pi^2)$ can not be larger than -1*, since no cancellation is possible due to the positivity of each term. Particularly we demonstrate here that there cannot be states behaving like $\mathcal{M} \sim O(1)$, $\Gamma \sim O(1)$. Furthermore, if M_i^2 is non-vanishing in the chiral limit, then we have $G_i/M_i^2 \sim O(1/N_c)$ or less. For these poles

with $G_i/M_i^2 \sim O(1/N_c)$, from the N_c dependence of the third term on the $r.h.s.$ of Eq. (13), one concludes that there should be at least one pole with $G \sim O(1/N_c)$, $M^2 \sim O(1)$. Such a pole corresponds to the normal resonance made of one quark and one anti-quark when M^2 is positive, that is $\mathcal{M} \sim O(1)$, $\Gamma \sim O(1/N_c)$. Such a result is not surprising at all since it is the the standard N_c counting rule for normal mesons. [8] However, our derivation is valuable since the S matrix pole's correspondence to the quark composites is not totally clear. The virtual pole in the IJ=20 channel, located at $s_0 = m_\pi^6/(16\pi^2 F_\pi^4) + O(m_\pi^8)$ on the s plane, is a living counter example. [9] If we consider only ordinary poles made of quarks and gluons, then $\mathcal{M} \sim O(1)$ as a fact of wave function normalization and $\Gamma \sim O(1/N_c)$ or less. The latter situation can not be excluded and may well happen in nature. For example, a glueball's decay width to $\pi\pi$ is $O(1/N_c^2)$.

The matching up to and including $(s - 4m_\pi^2)^2$ terms leads to the following three equations in the leading order of $O(1/N_c)$ expansion:

$$\sum_{n=i,r} \frac{4G_n m_\pi^2}{M_n^2-4m_\pi^2} = T_0^{\chi PT} - f_{L0}\,, \qquad (14)$$

$$\sum_{n=i,r} \frac{G_n M_n^2 m_\pi^2}{(M_n^2-4m_\pi^2)^2} = T_1^{\chi PT} - f_{L1}\,, \qquad (15)$$

$$\sum_{n=i,r} \frac{G_n M_n^2 m_\pi^4}{(M_n-4m_\pi^2)^3} = T_2^{\chi PT} - f_{L2}\,, \qquad (16)$$

where we have already replaced the partial wave T matrix on the $r.h.s$ of above equations by 1–loop chiral perturbation amplitudes: $T(s) = T^{\chi PT}(s) + O(p^6)$. Notice that in above equations the subscripts in $T^{\chi PT}$ and f_L imply the order of threshold expansion. It is not difficult to check using the results of chiral amplitudes that Eq. (14) and Eq. (15) are degenerate in the chiral limit. However, except for being helpful in determining the N_c counting of resonances, the equations (14) – (16) are of little use if one does not know how to calculate those f_{Ln} coefficients. Here we only point out that it is a good speculation to neglect numerically those crossed channel effects in the IJ=11 channel, since the left cut contribution is tiny in this channel [2]. Then Eqs. (14) – (16) read,

$$\sum_i \frac{G_{v,i}}{M_{v,i}^2} + \sum_r \frac{G_{v,r}}{M_{v,r}^2} = \frac{1}{96\pi F_\pi^2}\,, \qquad (17)$$

$$\sum_i \frac{G_{v,i}}{M_{v,i}^4} + \sum_r \frac{G_{v,r}}{M_{v,r}^4} = -\frac{L_3}{24\pi F_\pi^4}\,. \qquad (18)$$

Every parameter in above equations is understood as the corresponding value in the large N_c and chiral limit. Good agreement are found between two sides of the above two equations. Actually when neglecting the higher resonances (which are very small numerically) the Eq. (17) reproduces the well known KSFR relation. Left cut contributions in the IJ=20 and 00 channels are large, therefore it is not correct

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to neglect them at all. What we would like to emphasize here is that the σ pole must behave as $G/M^2 \sim O(N_c^{-1})$ if it contributes to Eq. (14) and/or (15) (or equivalently speaking, it contributes to F_π). It should further behave as $G \sim O(N_c^{-1})$, $M^2 \sim O(1)$, if it also contributes to Eq. (16) (i.e., it contributes to the LECs, the L_i parameters [7]). Related model dependent discussions on σ trajectory may be found in Ref. [10].

When obtaining the N_c counting rule for S matrix poles, we rely on the analyticity property of the partial wave S matrix. One of the most important result is that resonances with $M^2 \sim O(1)$ and $G \sim O(1)$ do *not* exist. The results do not depend much on whether we have analyticity on the whole cut plane. It is obtained based on two conditions: 1) all the S matrix poles appeared on the $l.h.s$ of Eq. (14) have the same sign, because they are all located in the Lehmann–Martin domain of analyticity; 2) the fact that the $r.h.s$ of Eq. (14) is $O(N_c^{-1})$. The latter condition follows from the fact that the T matrix itself is $O(N_c^{-1})$, and it remains to be true even if using Eq. (4$'$) instead of Eq. (4). It is of little interests to discuss the N_c dependence of poles located outside the Lehmann–Martin domain, if there are any.

The above discussions are only limited to $\pi\pi$ scatterings. However we believe the picture should also hold for any two pseudo-Goldstone boson scatterings, since the only difference comes from kinematics which should not waver the N_c counting rule. Taking $\pi\pi$, $\bar{K}K$ couple channel system for example, the analytic continuation of partial wave S matrices on different sheets are

$$S^{II} = \begin{pmatrix} \frac{1}{S_{11}} & \frac{iS_{12}}{S_{11}} \\ \frac{iS_{12}}{S_{11}} & \frac{\det S}{S_{11}} \end{pmatrix}, \quad S^{III} = \begin{pmatrix} \frac{S_{22}}{\det S} & \frac{-S_{12}}{\det S} \\ \frac{-S_{12}}{\det S} & \frac{S_{11}}{\det S} \end{pmatrix},$$
$$S^{IV} = \begin{pmatrix} \frac{\det S}{S_{22}} & \frac{-iS_{12}}{S_{22}} \\ \frac{-iS_{12}}{S_{22}} & \frac{1}{S_{22}} \end{pmatrix}. \tag{19}$$

From these expressions we realize that a third sheet pole in $S_{\pi\pi\to\pi\pi}$ which resides on the complex s plane when N_c large may only be cancelled by a nearby zero (a fourth sheet pole), i.e., they annihilate at $N_c = \infty$. But a third sheet pole in $T_{\pi\pi\to\pi\pi}$ is also a third sheet pole in $T_{\bar{K}K\to\bar{K}K}$. The wrongful N_c order of such a pole in the $T_{\bar{K}K\to\bar{K}K}$ amplitude may only be cancelled by a second sheet pole. However, the cancellation is impossible to occur, since we have demonstrated that no second sheet pole can reside on the complex s plane except real axis in the large N_c limit. Therefore we have just argued that no partial width of any resonance pole can be $O(1)$. The use of the analyticity condition is crucial in obtaining the N_c counting rule of meson resonances. In fact one can construct a unitary amplitude with correct N_c counting rule but violating the analyticity condition. In such an example, the second sheet pole resides on the complex s plane when N_c is large, but it annihilates with a first sheet pole when $N_c = \infty$.

However, in the $\pi\pi$, $\bar{K}K$ couple channel system, there exists the $f_0(980)$ state which may be interpreted as a $\bar{K}K$

molecular bound state. If it is indeed the case, then our previous discussions have to be reexamined since some of our results obtained depend on the hypothesis of the non-existence of bound state. If $f_0(980)$ is a $\bar{K}K$ bound state, it has $O(1)$ coupling to $\bar{K}K$ and $O(1/N_c)$ coupling to $\pi\pi$. Therefore it is a good approximation to neglect the $\pi\pi$ channel first. In such a case when there exists a bound state it can still be demonstrated that the cancellation between a bound state and resonance is impossible at the level of $O(1)$. The only possibility to restore the correct N_c order of the scattering amplitude is to allow an accompanying virtual state in association with the bound state. If both a_b and $a_v \sim O(1)$, but $a_b - a_v \sim O(1/N_c)$, then a correct N_c counting for the T matrix can be made. The cancellation works because,

$$S = \frac{(1 - i\rho a_b)(1 + i\rho a_v)}{(1 - i\rho a_b)(1 + i\rho a_v)} = \frac{\frac{4a_b a_v}{1 + a_b a_v} - s + i\rho s \frac{a_b - a_v}{1 + a_b a_v}}{\frac{4a_b a_v}{1 + a_b a_v} - s - i\rho s \frac{a_b - a_v}{1 + a_b a_v}}, \tag{20}$$

i.e., the net effect after the cancellation is very much like a normal resonance, but with a mass below the $\bar{K}K$ threshold, $M^2 = 4a_b a_v/(1 + a_b a_v) < 4m_K^2$. When coupling to $\pi\pi$ channel is opening, the bound state becomes a narrow second sheet pole (and hence described as the $f_0(980)$), and the virtual state becomes a 3rd sheet pole. Such a scenario is allowed within the present scheme, and there exists evidence that the data are better described by two poles near the $\bar{K}K$ threshold [11]. The discussions made above may be used to argue the non-existence of tetra-quark states, [12] except the bound/virtual state scenario just mentioned.

To conclude, with the aid of analyticity condition we observe that any S matrix pole trajectory on the s plane obtained by increasing N_c can only behave in one of the following three ways: 1) always remains on the real axis; 2) approaching real axis when $N_c \to \infty$; 3) moving to ∞ when $N_c \to \infty$. It is not totally clear whether the σ follows the second or the third trajectory though the second one is prefered. From the above observation we also conclude that in general tetra quark states do not exist.

[1] H. Q. Zheng *et al.*, Nucl. Phys. **A**733(2004)235;

[2] Z. Y. Zhou *et al.*, JHEP02(2005)043.

[3] A. Martin and F. Cheung, *Analyticity Properties and Bounds of the Scattering Amplitudes*, Gordon and Breach, New York 1970.

[4] B. R. Martin, D. Morgan and G. Shaw, *Pion Pion Interactions in Particle Physics*, Academic Press, London, 1976.

[5] The way we make the observation may not be totally un-ambiguous. Because rigorously speaking the Eq. (7) only works in a limited range of s. [4]

[6] P. D. B. Collins, *An Introduction to Regge Theory and High Energy Physics* Cambridge University Press, 1977.

[7] J. Gasser and H. Leutwyler, Nucl. Phys. B250(1985)465.

[8] E. Witten, Nucl. Phys. **B160** (1979) 57.

[9] Q. Ang *et al*., Commun. Theor. Phys. **36**(2001)563.

[10] Z. X. Sun et al, hep-ph/0411371; see also, J. Pelaez, Phys. Rev. Lett. **92**(2004)102001.

[11] D. Morgan and M. R. Pennington, Phys. Rev. **D48** (1993)1185; M. Locher, V. Markushin and H. Q. Zheng, Eur. Phys. J. **C4** (1998)317.

[12] H. Q. Zheng, talk given at MENU04, Beijing, China, 29 Aug - 4 Sep 2004; hep-ph/0411025.